

Therefore, the accuracy of calculations according to the selective-gray model depends primarily on the accuracy of calculation of radiation in the bands enumerated above.

#### NOTATION

$q$ , density of heat flux,  $W/m^2$ ;  $r$ ,  $\alpha$ , reflectivity and absorptivity, respectively;  $\psi$ , generalized angular coefficient;  $\lambda$ , wavelength,  $m$ ;  $C_1 = 3.74 \cdot 10^{-16} W/m^2$ ;  $C_2 = 1.4387 \cdot 10^{-2} m \cdot K$ ;  $\sigma_0 = 5.67 \cdot 10^{-8} W/(m^2 \cdot K^4)$ ;  $T$ , temperature,  $K$ ;  $(A)$ , matrix of coefficients of the unknowns in the system of zonal equations;  $(Q)$ ,  $(B)$ , column matrices of the unknowns and of the right-hand sides of the system of equations;  $k$ , absorption coefficient,  $m^{-1} \cdot atm^{-1}$ ;  $k_{\Delta\lambda}$ , total absorption coefficient in the band,  $m^{-1}$ ;  $\alpha$ , index of absorption,  $m^{-2} \cdot atm^{-1}$ ;  $\omega$ , wave number,  $m^{-1}$ ;  $p$ , pressure,  $N/m^2$ ; indices:  $p$ , resultant;  $c$ , natural value or eigenvalue;  $V$ , volume;  $F$ , area.

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#### DETERMINATION OF THE THERMAL CONDUCTIVITY OF ANISOTROPIC MEDIA

#### ON THE BASIS OF THE SCANNING METHOD: THEORETICAL MODELS AND EXPERIMENTAL IMPLEMENTATION OF THE METHOD

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Procedures for determining the thermal conductivity of anisotropic media are developed on the basis of an analysis of the temperature fields in anisotropic media under the influence of moving energy sources.

In the multivariety of procedures based on the scanning method for measuring the thermal conductivity of anisotropic media, procedures that utilize the solutions for a point energy source and a combination of a line source and a point source are particularly valuable in practice. Here we discuss theoretical models of the proposed procedures, making use of the analytical relations obtained in the first part of the study [1] for the temperature fields of moving energy sources in anisotropic media.

#### THEORETICAL MODEL OF THE PROCEDURE BASED ON A POINT ENERGY SOURCE

On the investigated anisotropic sample with two noncoplanar plane surface we choose three arbitrary noncolinear directions, which are not in the same plane and are specified by unit vectors  $\mathbf{n}_1$ ,  $\mathbf{n}_2$ , and  $\mathbf{n}_3$ . The surfaces of the sample are scanned successively along the selected directions by a continuously acting point energy source and a temperature sensor, which moves along the line of heating at the speed of the source, following it at a distance  $d$  (Fig. 1). The maximum excess temperatures  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  recorded on the heated surfaces of the sample, according to Eq. (11) (in the first part of the study [1]), are equal to

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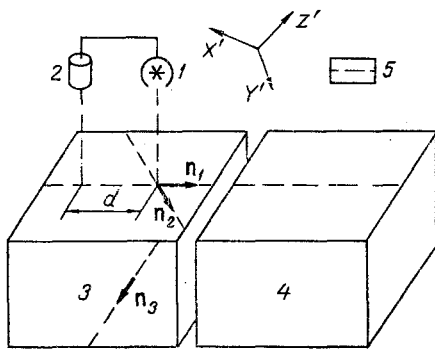


Fig. 1

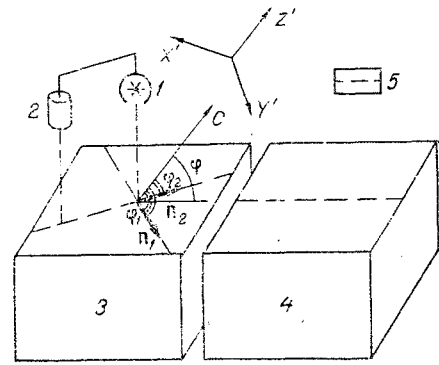


Fig. 2

Fig. 1. Diagram of the procedure for investigations using a point source. 1) Energy source; 2) temperature sensor; 3) anisotropic sample; 4) standard;  $\mathbf{n}_1$ ,  $\mathbf{n}_2$ ,  $\mathbf{n}_3$  are the sample scanning directions; 5) lines of heating of the sample and the standard.

Fig. 2. Diagram of the procedure for investigations of samples when two of the principal thermal conductivities are equal. 1) energy source; 2) temperature sensor; 3) anisotropic sample; 4) standard;  $\mathbf{n}_1$  and  $\mathbf{n}_2$  are the sample scanning directions; 5) lines of heating of the sample and the standard.

$$\begin{aligned}\theta_1 &= \frac{W}{2\pi d (\lambda_1 \lambda_2 \lambda_3)^{1/2}} \left( \frac{\alpha_1^2}{\lambda_1} + \frac{\beta_1^2}{\lambda_2} + \frac{\gamma_1^2}{\lambda_3} \right)^{-1/2}, \\ \theta_2 &= \frac{W}{2\pi d (\lambda_1 \lambda_2 \lambda_3)^{1/2}} \left( \frac{\alpha_2^2}{\lambda_1} + \frac{\beta_2^2}{\lambda_2} + \frac{\gamma_2^2}{\lambda_3} \right)^{-1/2}, \\ \theta_3 &= \frac{W}{2\pi d (\lambda_1 \lambda_2 \lambda_3)^{1/2}} \left( \frac{\alpha_3^2}{\lambda_1} + \frac{\beta_3^2}{\lambda_2} + \frac{\gamma_3^2}{\lambda_3} \right)^{-1/2},\end{aligned}\quad (20)$$

where  $\alpha_m$ ,  $\beta_m$ , and  $\gamma_m$  ( $m = 1, 2, 3$ ) are the direction cosines of the respective vectors  $\mathbf{n}_1$ ,  $\mathbf{n}_2$ , and  $\mathbf{n}_3$  in the  $X'Y'Z'$  coordinate system. The surface of a standard with thermal conductivity  $\lambda_{st}$ , which is placed in line with the investigated sample, is also scanned by the energy source and the temperature sensor during the scanning of the investigated sample (in each of the directions  $\mathbf{n}_1$ ,  $\mathbf{n}_2$ , and  $\mathbf{n}_3$ ). The maximum excess temperature  $\theta_{st}$  recorded by the sensor on the surface of the standard, according to [2], is given by the relation

$$\theta_{st} = W (2\pi d \lambda_{st})^{-1}. \quad (21)$$

If the source power  $W$  and the measurement baseline  $d$  are constant, the principal thermal conductivities  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  of the investigated sample are determined from a system of equations derived on the basis of Eqs. (20) and (21):

$$\begin{aligned}\Lambda_1^2 &\equiv \left( \lambda_{st} \frac{\theta_{st}}{\theta_1} \right)^2 = \alpha_1^2 \lambda_2 \lambda_3 + \beta_1^2 \lambda_1 \lambda_3 + \gamma_1^2 \lambda_1 \lambda_2, \\ \Lambda_2^2 &\equiv \left( \lambda_{st} \frac{\theta_{st}}{\theta_2} \right)^2 = \alpha_2^2 \lambda_2 \lambda_3 + \beta_2^2 \lambda_1 \lambda_3 + \gamma_2^2 \lambda_1 \lambda_2, \\ \Lambda_3^2 &\equiv \left( \lambda_{st} \frac{\theta_{st}}{\theta_3} \right)^2 = \alpha_3^2 \lambda_2 \lambda_3 + \beta_3^2 \lambda_1 \lambda_3 + \gamma_3^2 \lambda_1 \lambda_2,\end{aligned}\quad (22)$$

according to the equations

$$\lambda_1 = \left( \frac{D_2 D_3}{D_1 \det T} \right)^{1/2}, \quad \lambda_2 = \left( \frac{D_1 D_3}{D_2 \det T} \right)^{1/2}, \quad \lambda_3 = \left( \frac{D_1 D_2}{D_3 \det T} \right)^{1/2}, \quad (23)$$

where

$$D_k = \sum_{i=1}^3 T_{ik} \Lambda_i^2 \quad (k = 1, 2, 3), \quad (24)$$

and  $T_{ik}$  is the minor of the element  $t_{ik}$  in the determinant

$$\det T \equiv \det \{t_{ik}\} = \begin{vmatrix} \alpha_1^2 & \beta_1^2 & \gamma_1^2 \\ \alpha_2^2 & \beta_2^2 & \gamma_2^2 \\ \alpha_3^2 & \beta_3^2 & \gamma_3^2 \end{vmatrix}. \quad (25)$$

The proposed procedure for determination of the thermal conductivity of anisotropic media thus entails:

scanning of two noncoplanar surfaces of the sample in three noncollinear directions by a rigid connected point energy source and a temperature sensor, where each time the surface of a standard placed in line with the investigated sample is scanned;

determination of the orientation of the sample scanning directions relative to the principal heat-conduction axes of the sample;

calculation of the required unknowns  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  according to Eqs. (23) and (24).

Also of practical interest is the case in which two of the three principal thermal conductivity are equal, i.e.,  $\lambda_1 = \lambda_2 = \lambda_a$ ,  $\lambda_3 = \lambda_c$ . This happens, e.g., for single crystals of minerals of medium symmetry. In order to determine the principal thermal conductivities of the medium in this case, it is sufficient to scan one surface, which forms an angle  $\varphi$  other than  $90^\circ$  with the principal thermal conductivity axis  $C \parallel Z'$ , along two arbitrary noncollinear directions  $\mathbf{n}_1$  and  $\mathbf{n}_2$ , which form angles  $\varphi_1$  and  $\varphi_2$  with the C axis (Fig. 2).

Indeed, it follows from Eq. (22) for  $\lambda_1 = \lambda_2 = \lambda_a$ ,  $\lambda_3 = \lambda_c$  that

$$\Lambda_1^2 \equiv \left( \lambda_{st} \frac{\theta_{st}}{\theta_1} \right)^2 = \lambda_a \lambda_c \sin^2 \varphi_1 + \lambda_a^2 \cos^2 \varphi_1, \quad (26)$$

$$\Lambda_2^2 \equiv \left( \lambda_{st} \frac{\theta_{st}}{\theta_2} \right)^2 = \lambda_a \lambda_c \sin^2 \varphi_2 + \lambda_a^2 \cos^2 \varphi_2.$$

Solving the system of equations (26), we obtain

$$\lambda_a = \left( \frac{\Lambda_2^2 \sin \varphi_1 - \Lambda_1^2 \sin \varphi_2}{\cos^2 \varphi_2 - \cos^2 \varphi_1} \right)^{1/2}, \quad (27)$$

$$\lambda_c = \frac{\Lambda_1^2 \cos^2 \varphi_2 - \Lambda_2^2 \cos^2 \varphi_1}{[(\cos^2 \varphi_2 - \cos^2 \varphi_1)(\Lambda_2^2 \sin^2 \varphi_1 - \Lambda_1^2 \sin^2 \varphi_2)]^{1/2}}.$$

A necessary condition for the determination of the principal thermal conductivities in the implementation of the procedure using a point energy source is the existence of two noncoplanar plane surfaces on the investigated samples in the general case. A combination of a linear source and a point source can be used to determine  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  even if the sample has only one plane surface. This affords additional possibilities for nondestructive investigations of unconventional objects.

#### THEORETICAL STUDY OF THE PROCEDURE BASED ON A COMBINATION OF A POINT SOURCE AND A LINE SOURCE OF ENERGY

To determine the principal thermal conductivities  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  of an anisotropic sample having one plane boundary source perpendicular to, e.g., the  $Z'$  axis (Fig. 3), we scan the given surface in two noncollinear directions  $\mathbf{n}_1$  and  $\mathbf{n}_2$  by a rigidly connected point energy source and temperature sensor, which moves along the line of heating at a distance  $d$  behind the source (Fig. 3a). If the surface of the standard is also scanned in each of the same directions during heating and recording of the maximum excess temperature  $\theta_1$  and  $\theta_2$  of the surface of the investigated sample, we obtain the following expressions for the constants  $W$  and  $d$  according to Eqs. (20):

$$\Lambda_1^2 = \lambda_2 \lambda_3 \alpha_1^2 + \lambda_1 \lambda_3 \beta_1^2, \quad \Lambda_2^2 = \lambda_2 \lambda_3 \alpha_2^2 + \lambda_1 \lambda_3 \beta_2^2. \quad (28)$$

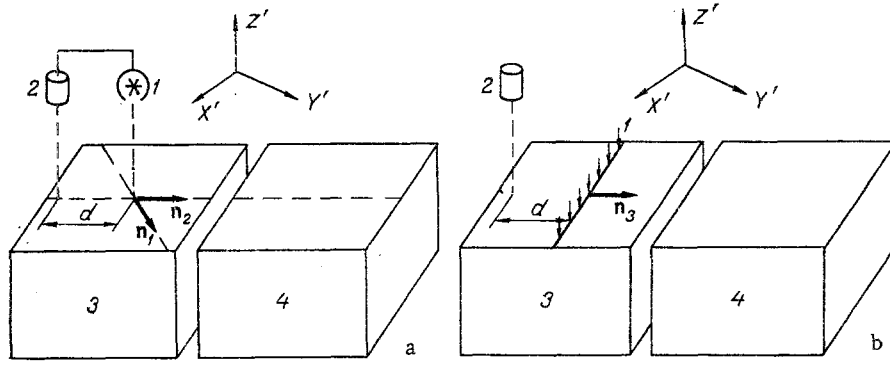


Fig. 3. Diagram of the procedure for investigations based on a combination of energy sources. a) Energy source; 2) temperature sensor; 3) anisotropic sample; 4) standard;  $\mathbf{n}_1$ ,  $\mathbf{n}_2$ , and  $\mathbf{n}_3$  are the sample scanning directions.

We then scan the surfaces of the investigated sample and the standard in an arbitrary direction  $\mathbf{n}_3$  by a rigidly connected line energy source and temperature sensor (Fig. 3b). The maximum excess temperatures recorded on the surfaces of the sample and the standard in this case ( $\theta_3$  and  $\theta_{st}^*$ , respectively) are determined, according to Eq. (19), from the relations

$$\theta_3 = q (\pi \lambda_3 c \rho v d)^{-1/2}, \quad (29)$$

$$\theta_{st}^* = q (\pi \lambda_{st} (c\rho)_{st} v d)^{-1/2}. \quad (30)$$

For constant values of  $q$  and  $d$  and a known volume specific heat of the investigated sample, from Eqs. (28)-(30) we obtain analytical equations for determining the principal thermal conductivities:

$$\lambda_1 = \lambda_3^{-1} \frac{\Lambda_2^2 \alpha_1^2 - \Lambda_1^2 \alpha_2^2}{\alpha_1^2 \beta_2^2 - \alpha_2^2 \beta_1^2}, \quad \lambda_2 = \lambda_3^{-1} \frac{\Lambda_1^2 \beta_2^2 - \Lambda_2^2 \beta_1^2}{\alpha_1^2 \beta_2^2 - \alpha_2^2 \beta_1^2}, \quad (31)$$

$$\lambda_3 = \lambda_{st} \frac{(c\rho)_{st}}{c\rho} \left( \frac{\theta_{st}^*}{\theta_3} \right)^2.$$

An analysis of the above-described theoretical models and the experimental results have shown that these procedures generate different random errors of determination of the principal thermal conductivities, depending on the choice of directions in which to scan the samples. It is important in this regard to investigate the proposed measurement procedures in order to determine the optimum sample scanning directions, i.e., the directions in which the best metrological indices are attained.

We carry out such investigations for the point-source procedure, which is the most facile from the point of view of practical implementation. To simplify the analysis, we consider the case in which two of the three principal thermal conductivities are equal, i.e.,  $\lambda_1 = \lambda_2 = \lambda_a$ ,  $\lambda_3 = \lambda_c$  (see Fig. 2). The primary source of random error in the determination of the principal thermal conductivities of  $\lambda_a$  and  $\lambda_c$  of the investigated sample are errors of recording of the sample and standard temperatures, i.e., essentially the random errors of the experimentally determined quantities  $\Lambda_1$  and  $\Lambda_2$ . Differentiating Eqs. (27) with respect to  $\Lambda_1$  and  $\Lambda_2$  and assuming that the relative random errors of determination of  $\Lambda_1$  and  $\Lambda_2$  are equal (i.e.,  $\delta\Lambda_1 = \delta\Lambda_2 = \delta\Lambda$ ), we obtain expressions for the relative random errors  $\delta\lambda_a$  and  $\delta\lambda_c$  of determination of the principal thermal conductivities:

$$\delta\lambda_a = (A^2 + B^2)^{1/2} \delta\Lambda, \quad \delta\lambda_c = (C^2 + D^2)^{1/2} \delta\Lambda, \quad (32)$$

where

$$A = \frac{\sin^2 \varphi_1 [(\lambda_c/\lambda_a) \sin^2 \varphi_2 + \cos^2 \varphi_2]}{\cos^2 \varphi_2 \sin^2 \varphi_1 - \cos^2 \varphi_1 \sin^2 \varphi_2};$$

$$B = \frac{\sin^2 \varphi_2 [(\lambda_c/\lambda_a) \sin^2 \varphi_1 + \cos^2 \varphi_1]}{\cos^2 \varphi_2 \sin^2 \varphi_1 - \cos^2 \varphi_1 \sin^2 \varphi_2};$$

$$C = \frac{[\sin^2 \varphi_1 + (\lambda_a/\lambda_c) \cos^2 \varphi_1][(\lambda_c/\lambda_a) \sin^2 \varphi_2 + 2 \cos^2 \varphi_2]}{\cos^2 \varphi_2 \sin^2 \varphi_1 - \cos^2 \varphi_1 \sin^2 \varphi_2};$$

$$D = \frac{[\sin^2 \varphi_2 + (\lambda_a/\lambda_c) \cos^2 \varphi_2][(\lambda_c/\lambda_a) \sin^2 \varphi_1 + 2 \cos^2 \varphi_1]}{\cos^2 \varphi_2 \sin^2 \varphi_1 - \cos^2 \varphi_1 \sin^2 \varphi_2}.$$

Analyzing Eqs. (32) and (33), we readily show that for  $\varphi_1 > \varphi_2$  the quantities  $\delta\lambda_a$  and  $\delta\lambda_c$  are nonnegative decreasing functions of the angle  $\varphi_1$ . It is evident from Fig. 2 that the angles formed by all possible scanning lines on the surface of the sample with its C axis fall within the interval from  $\varphi$  to  $90^\circ$ , so that for any fixed value of  $\varphi_2$  the quantities  $\delta\lambda_a$  and  $\delta\lambda_c$  acquire the minimum values at  $\varphi_1 = 90^\circ$ , i.e., when the scanning direction on the sample surface  $\mathbf{n}_1$  is perpendicular to the C axis. Substituting  $\varphi_1 = 90^\circ$  in relation (33), we obtain

$$A = D = 1 + (\lambda_c/\lambda_a) \operatorname{tg}^2 \varphi_2, \quad B = (\lambda_c/\lambda_a) \operatorname{tg}^2 \varphi_2, \quad C = 2 + (\lambda_c/\lambda_a) \operatorname{tg}^2 \varphi_2. \quad (34)$$

It follows from Eqs. (34) that the quantities  $\delta\lambda_a$  and  $\delta\lambda_c$  are nonnegative increasing functions of the angle  $\varphi_2$  for  $\varphi_1 = 90^\circ$ . Consequently,  $\delta\lambda_a$  and  $\delta\lambda_c$  assume minimum values for the minimum possible value of the angle  $\varphi_2$ , which is equal to  $\varphi$  in the given situation (Fig. 2).

Thus, to minimize the random errors of determination of the principal thermal conductivities of anisotropic media, it is necessary to scan the surface of the sample in the two optimum directions: One direction must be perpendicular to the principal C axis of heat conduction, and the second must coincide with the projection of this axis onto the heated surface of the investigated sample.

It follows from Eqs. (33) and (34) that the best metrological indices of the procedure for determining the thermal conductivity of anisotropic media on the basis of a point energy source are attained when  $\varphi_1 = 90^\circ$  and  $\varphi_2 = 0$ , i.e., when one of the scanning directions is perpendicular to the C axis and the other coincides with it. The analytical relations for determining the principal thermal conductivities acquire the simplest form in this case:

$$\lambda_a = \Lambda_2, \quad \lambda_c = \Lambda_1^2/\Lambda_2, \quad (35)$$

where, according to Eqs. (32) and (34), the quantity  $\delta\lambda_a$  does not exceed the random error of determination of the thermal conductivity of isotropic media [3], and  $\delta\lambda_c = \sqrt{5}\delta\lambda_a$ .

Generalizing the results, we can show that the optimum sample scanning directions in the investigation of anisotropic media with principal thermal conductivities  $\lambda_1 \neq \lambda_2 \neq \lambda_3$  coincide with the principal heat-conduction axes of the investigated media. According to Eqs. (22) and (23), the quantities  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  in this case are determined from the equations

$$\lambda_1 = \frac{\Lambda_2 \Lambda_3}{\Lambda_1}, \quad \lambda_2 = \frac{\Lambda_1 \Lambda_3}{\Lambda_2}, \quad \lambda_3 = \frac{\Lambda_1 \Lambda_2}{\Lambda_3}, \quad (36)$$

where the random errors of determination of  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  are  $\sqrt{3}$  times the random error of measurement of the thermal conductivity of isotropic media.

The practical implementation of the proposed method entails the application of noncontacting devices for heating the samples and recording their temperatures. The experimental arrangement for determining the thermal conductivity of anisotropic materials [3] includes an optical point source of thermal energy, a noncontacting temperature sensor responsive to thermal radiation from the heated surfaces of the samples, and an electromechanical system with a movable platform, on which the investigated samples and the standard are placed during the heating and measurement process. The energy source in this arrangement is a continuous-wave laser with a radiation wavelength of  $10.6 \mu\text{m}$  and a power of 3–5 W in the beam spot. A radiometer with a bandwidth of 2 to  $20 \mu\text{m}$  is used for the noncontacting temperature measurements.

The apparatus is standardized by means of standard thermal conductivity samples (optical glasses KV, K8, LK5, titanium alloy VT-6, stainless steel 12Kh18N10T) and certified samples of white marble. The standards have been certified at the All-Union Scientific-Research Institute of Metrology. The total error of the thermal conductivity of the standards according to the certification results is not greater than 2.5%.

TABLE 1. Thermal Conductivity of Single Crystals (T = 300 K)

Mineral, site	Symmetry	$\lambda_c$ (W/m·K)	$\lambda_a$ (W/m·K)	$\lambda_b$ (W/m·K)	$\lambda_a/\lambda_c$
Pyrrhotite, unnamed site, Yakutia	Hexagonal	3,71	3,43	3,43	0,92
Rutile, Georgia, USA	Tetragonal	5,92	4,38	4,38	0,74
Quartz (rock crystal), Yakutia	Trigonal	10,8	6,10	6,10	0,56
Quartz (morion), Zabaikale	»	10,8	6,10	6,10	0,56
Quartz (false topaz)	»	10,8	6,10	6,10	0,56
Quartz (synthetic)	»	10,8	6,10	6,10	0,56
Vesuvian, Tuva ASSR	Tetragonal	2,54	2,43	2,43	0,96
Vesuvian, Vilyui River, Yakutia	»	2,30	2,07	2,07	0,90
Vesuvian, Vilyui River, Yakutia	»	2,18	2,02	2,02	0,93
Beryl, Eastern Siberia	Hexagonal	4,44	3,75	3,75	0,84
Beryl, Eastern Siberia	»	4,17	3,88	3,88	0,93
Tourmaline (schorlite), Bavaria	»	2,77	4,08	4,08	1,47
Tourmaline (schorlite), Greenland	»	3,61	4,64	4,64	1,29
Muscovite, Ilmen Mts., Urals	Monoclinic	1,03	3,80	3,80	3,69
Biotite, Vishnev Mts., Urals	»	1,30	2,61	2,61	2,01
Chlorite (peninite), Nazem Mts., Urals	Monoclinic	1,38	11,1	11,1	8,0
Scapolite, Slyudyanka site, Zabaikal	Tetragonal	1,59	1,34	1,34	0,84
Calcite (Iceland spar), Nizhnei Tunguski region	Trigonal	3,50	3,21	3,21	0,92
Apatite, Slyudyanka site, Zabaikale	Hexagonal	1,63	1,51	1,51	0,93
Apatite, Slyudyanka site, Zabaikale	»	1,72	1,47	1,47	0,85
Apatite, Slyudyanka site, Zabaikale	»	1,76	1,60	1,60	0,91

The apparatus developed here can be used to determine the principal thermal conductivities of anisotropic materials in the range of 1-15 W/m K within 7% error limits. The systematic error component associated with the error of the values for the standards is not greater than 2.5%, and the random component is  $\pm 4.5\%$  with confidence coefficient 0.95. The high speed of the measurement process (15-20 samples per hour) makes it possible to decrease the random error component significantly as a result of repeated measurements.

We have used the proposed method to determine the principal thermal conductivities of 21 single crystals of 12 different materials among those for which reliable data on the anisotropy of the thermal conductivity have not been published in the literature. For the investigations we chose crystals with symmetries characterized by equal values of two of the principal thermal conductivities (i.e.,  $\lambda_1 = \lambda_2 = \lambda_a$ ,  $\lambda_3 = \lambda_c$ ). This enabled us to determine the principal thermal conductivities from one face of the crystal, using the simplest procedure for technical implementation with a point energy source. The results of the investigations are summarized in Table 1. Because of averaging of the data from the repeated measurements, the error of the tabulated values of  $\lambda_a$  does not exceed 3%, and the error of  $\lambda_c$  does not exceed 5%.

Thus, the foregoing results reinforce future prospects for the application of the scanning method for nondestructive investigations of the thermal conductivity of anisotropic materials.

#### NOTATION

$\theta$ , maximum excess temperature;  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ ,  $\lambda_a$ ,  $\lambda_c$ , principal thermal conductivities of anisotropic medium;  $\mathbf{n}_1$ ,  $\mathbf{n}_2$ ,  $\mathbf{n}_3$ , unit vectors in the sample scanning directions;  $\alpha$ ,  $\beta$ ,  $\gamma$ , direction cosines of vector  $\mathbf{n}$  in coordinates X'Y'Z', whose axes coincide with the principal heat-conduction axes of the medium; W, power of energy source; q, power density of source; d, distance from energy source to temperature sensor;  $T_{ik}$ , minor of matrix;  $c_p$ , volume specific heat; v, speed of moving energy source.

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